Indian Statistical Institute, Bangalore
B. Math. Third Year
Second Semester - Differential Equations

Mid-Semester Exam   Duration : 3 hours   Date : Feb 22, 2017

Answer any five, each question carries 8 marks, total marks: 40

1. (a) Find a solution of $xy' = y + 2xe^{\frac{-y}{x}}$ *(Marks: 3)*.

   (b) Prove that $Mdx + Ndy = 0$ is exact if and only if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ and use it to solve $(\sin x \sin y - xe^y)dy = (e^y + \cos x \cos y)dx$.

2. (a) Let y_1 and y_2 be two linearly independent solutions of $y'' + P(x)y' + Q(x)y = 0$ on $[a, b]$. Show that $P = \frac{y_2 y_1'' - y_1 y_2''}{W(y_1, y_2)}$ and $Q = \frac{y_1' y_2'' - y_1'' y_2'}{W(y_1, y_2)}$ *(Marks: 5)*.

   (b) Let y and y' be linearly independent solutions of $y'' + P(x)y' + Q(x)y = 0$ on $[a, b]$. Suppose y'' is also a solution. Prove that P and Q are constants.

3. (a) Reduce $x^2 y'' + xpy' + qy = 0$ to a linear equation with constant coefficients and use it to solve $x^2 y'' + 2xy' - 12y = 0$.

   (b) Solve $y'' + y = 2\cos x$ *(Marks: 3)*.

4. (a) Solve the system $x' = 3x - 4y$ and $y' = x - y$ *(Marks: 4)*.

   (b) Let y be a nonzero solution of $y'' + P(x)y' + Q(x)y = 0$ on $[a, b]$. Prove that $\{x \in [a, b] \mid y(x) = 0\}$ is a finite set.

5. (a) Solve $(1+x)y' = py$, $y(0) = 1$ and prove $(1+x)^p = 1 + \sum_{n \geq 1} \frac{p(p-1)\cdots(p-(n-1))}{n!} x^n$ for $|x| < 1$ *(Marks: 4)*.

   (b) Solve $(1 + x^2)y'' + 2xy' - 2y = 0$ in terms of power series of x.

6. (a) Solve $y'' - 2xy' + 2py = 0$ and show that any solution is analytic on $\mathbb{R}$.

   (b) Solve $y'' + xy = 0$ using power series method *(Marks: 4)*.

7. (a) Find two independent Frobenius series solutions of $2xy'' + (3 - x)y' - y = 0$.

   (b) Prove that $\cos x = \lim_{a \to \infty} F(a, a, \frac{1}{2}, \frac{-x^2}{4a^2})$ *(Marks: 3)*.